

FRACTIONAL DIFFUSION EQUATIONS

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Abstract

The following topics are reviewed with emphasis on their influence in inverse theory.

Brief Historical Notes: From 1695 to present times. First fractional inverse problem.

Fractional Derivatives: Fractional derivatives as ill-posed problems. Regularization. Stable numerical evaluation of fractional derivatives. Numerical results. Examples.

Half Time Fractional Diffusion Equation: First fractional diffusion inverse problem. Restrictions. Need for stabilization. Numerical method. Error estimates.

Classification of Fractional Diffusion Equations: Space fractional diffusion equations (superdiffusion). Different time fractional diffusion equations (subdiffusion) depending on the location of the time fractional derivative. Equivalent formulations according to the type of time fractional derivative. Finite difference explicit solver. Numerical examples. Characterization of slow diffusion. Asymptotic behavior.

Time Fractional Inverse Heat Conduction Problem: Generalization of the standard inverse heat conduction problem. Degree of ill-posedness. Regularization. Adaptive space marching mollified algorithm. Error analysis. Generating Cauchy data for the inverse problem. Numerical examples.